Incentives for Electronic Coupon Systems

Jussi Kangasharju
jussi@tk.informatik.tu-darmstadt.de

Andreas Heinemann
aheine@tk.informatik.tu-darmstadt.de

Department of Computer Science, Darmstadt University of Technology
Hochschulstraße 10, D-64289 Darmstadt, Germany

ABSTRACT

Ad hoc networks and peer-to-peer systems typically require many users to participate, in order to leverage the full benefits of the system. In this paper we examine an electronic coupon system, where providers send out coupons, which are passed from user to user. The incentive for users to participate is that they receive bonus points for each redemption of the coupon. The contributions of this paper are two-fold. First, we define a general model for such bonus point -based coupon schemes and derive an optimal strategy which allows each user to determine how many bonus points she should ask for when passing the coupon. Second, we show that our optimal strategy is extremely robust against all other user strategies, and that there is a strong incentive for users to follow our optimal strategy.

Categories and Subject Descriptors

C.2.8.d [**Computer Systems Organization**]: Communication: Mobile Computing: Mobile Environments

General Terms

Performance, Economics

Keywords

Incentives, Ad Hoc Networks, Electronic Coupons

1. INTRODUCTION

Many systems, such as ad hoc networks and peer-to-peer systems, require many users to participate, in order to leverage the full benefits of the system. However, in many cases, users may be required to provide *their own resources* (e.g., memory, bandwidth, battery power) for others to use, without getting any direct benefit from that. Work on mobile ad hoc networks has proposed several *incentive schemes*, see for example [2, 3], to encourage user participation.

In this paper, we examine an electronic coupon system, where a provider sends out a coupon which is passed from user to user, until a user finally redeems the coupon (typically, purchases the advertised product). In order to encourage user participation, the provider gives out bonus points (e.g., frequent flyer miles) to all users who participated in passing the coupon to the redeemer.

The contributions of this paper are two-fold. First, we define a general model for such bonus point -based coupon schemes and derive an optimal strategy which allows each user to determine how many bonus points she should ask for when passing the coupon. Second, we show that our optimal strategy is extremely robust against all other user strategies, and that there is a strong incentive for users to follow our optimal strategy.

2. ELECTRONIC COUPON MODEL

The system we consider is adPASS [7], which spreads electronic coupons among interested users in mobile environments. A provider sends out coupons which advertise some products via an information sprinkler installed in the shop. Users carry mobile devices which store the coupons according to the user's preferences. Later, when a user A who has coupons meets another user B with similar interests, A can pass the coupon on to B. As a reward for this, the provider has allocated some number of bonus points to the coupon and A is allowed to claim some of these points for herself. B can also pass the coupon on, or go to the shop to redeem the coupon. When some user goes to redeem the coupon, *all* users who were involved in the chain this coupon took from the provider to the redeeming user will get the bonus points they took when they passed the coupon on. If nobody redeems the coupon, no bonus points are given out.

2.1 Bonus Point Model

If a user redeems the coupon, e.g., purchases the advertized product, all the users who participated in passing the coupon from the provider to the actual buyer get the points they have taken. We call this sequence of users the *passing chain*, or chain. A chain always starts from the provider and ends in a user who redeems the coupon. For a user A, we call all the users between the provider and A as *upstream users* and all users after A in the chain are *downstream users*.

The rules for passing the coupons to others are as follows:

1. The provider sets the initial number of points.
2. Each user must take at least 1 point when she passes the coupon onwards.
3. If only 1 point remains, the coupon cannot be passed.

MobiShare'06, September 25, 2006, Los Angeles, California, USA.

4. If several downstream users redeem the same coupon, upstream users get the points for *each* redemption.

5. If a user redeems a coupon, she can get points for the same coupon if she had passed it before redeeming it.

2.2 Formal Model

Let B be the number of points in a coupon that user A receives. User A might get the coupon from the provider or from another user. For A, there is no difference between these cases and only the remaining points B matter.

Assume that user A decides to take b points, with $1 \leq b < B$. We also assume that the probability that any user redeems the coupon is p. For simplicity, we assume that this probability is the same for all users and that the users are independent from each other. Furthermore, we assume that the redemption probability is independent of the number of points remaining in the coupon.

Since each user has to take at least 1 point from the coupon, the length of the passing chain is finite. Denote the length of the downstream chain from user A as N. See Section 3 on how the length of the chain can be calculated.

Then, the expected number of points A can get from all the redemptions in her downstream chain is:

$$\begin{aligned} E[\text{points if } b \text{ taken}] &= \sum_{i=1}^{N} bi \binom{N}{i} p^i (1-p)^{N-i} \\ &= bNp \end{aligned} \quad (1)$$

Note that the redemption probability p has no effect on b and N which are determined by the strategy of the user. Hence, our assumption that p is the same for all users has no effect on the user behavior.

3. STRATEGIES

We assume that the goal of a user is to maximize the number of bonus points she earns. As can be seen from equation (1), the redemption probability does not affect the strategy of the user, since the points taken determine the length of the downstream chain.

Consider the case where there is only 1 point left in the coupon. In this case, the coupon cannot be passed on and the chain ends. Similarly, when there are 2 points left, the only possible solution is to take 1 point and pass 1 point on. Since users try to maximize their own benefit, we assume that the user will try to pass the coupon onwards.

With 3 points remaining, a user A can either take 1 or 2 points. If user A takes 2 points, then only 1 point remains and the chain after the current user has length 1. If user A takes 1 point, then two points remain, and the above strategy for 2 points results in a chain of length 2. In both cases, the expected number of gained points given by equation (1) is $2p$. Since the two strategies have the same expected number of points gained, they are equal for that user.

With 4 points remaining, a user A has three possibilities: take 3, 2, or 1 points. We can calculate that the expected number of gained points are $3p$, $4p$ and $3p$, respectively.[1]

[1]The case where A takes only 1 point divides into two possibilities, depending on whether the next user takes 1 or 2 points. These two possibilities give the expected number of gained points as $3p$ and $2p$.

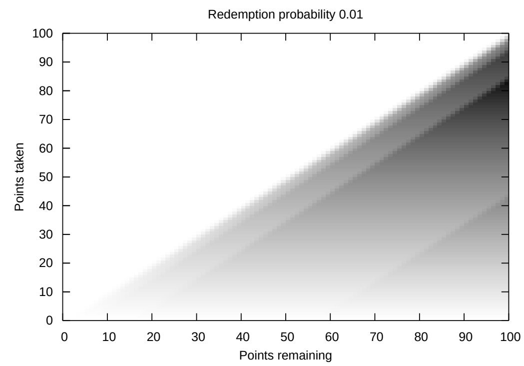

Figure 1: Optimal strategy for 1–100 points

The strategy "Take 2" dominates, and the optimal strategy for 4 remaining points is to take 2 of them.

In a similar way, we can continue with larger number of points remaining, and we can compute the optimal strategies for any given value of B. Figure 1 shows the optimal strategy for 1–100 points remaining.

In these figures, the x-axis shows how many points are remaining in the coupon when the user receives it, and the y-axis shows the possible amounts of points the user can take for herself. The color at a point (x, y) indicates the expected number of bonus points gained by the user, assuming she took y points from the remaining x points. Darker colors indicate higher amounts of gained points.

As we can see, the optimal choice of points is typically to take about 60–80% of the remaining points. This is especially clearly visible on the right side of Figure 1, where the remaining number of points is large.

4. EFFECT OF USER BEHAVIOR

The optimal strategy assumes that all users follow the optimal strategy. We now investigate the sensitivity of the optimal strategy to other possible strategies followed by other users.

We consider 6 different types of users, defined as follows:

- Rational: Follows the optimal strategy (take *opt* points).

- Greedy: Takes more points than the optimal strategy says. A *pure greedy* user always leaves only 1 point.

- Altruistic: Takes less points than the optimal strategy says. A *pure altruistic* user always takes only 1 point.

- Random: Takes a random amount of points, uniformly distributed between 1 and $B - 1$.

For the normal greedy and altruistic strategies, we assume that the users take a random amount of points, uniformly distributed in the appropriate interval ($]opt, max - 1]$ for greedy and $[1, opt[$ for altruistic).

We compare each of the 6 user types pairwise with all the other types. We assume a population which consists entirely of the given type and we investigate what happens when a single user from another type comes into play.

My strategy	Opponent's strategy R	G	A	RND	P-G	P-A
Rational (R)	**4.20**	**2.18**	**5.23**	**4.30**	1.68	13.44
Greedy (G)	3.23	1.99	3.88	4.04	**1.77**	7.73
Altruistic (A)	2.37	1.11	3.15	2.35	0.81	**18.09**
Random (RND)	2.37	1.29	3.20	2.90	0.95	16.59
Pure greedy (P-G)	0.99	0.99	0.99	0.99	0.99	0.99
Pure altruistic (P-A)	0.06	0.03	0.08	0.07	0.02	0.99

Table 1: Redemption probability $p = 0.01$ and initial bonus points $B = 100$. Best strategy shown in bold.

The results are shown in Table 1. We show the amount of gained bonus points by the user type on each row against a population indicated by the column. The values are averaged over 200 individual runs. In the simulation, we had one user of the type given on the row take as many points as her strategy dictates. Then she would pass the coupon on, and each user in the chain would use the "column-strategy" to decide how many points to take. We assumed that the coupon gets passed as long as there are points remaining.

The best strategy against the given population (column) is given by the bolded entries. The actual numerical values are specific to the parameters of the simulation; however, the ranking applies to all combinations of parameters.

The rational user gets the highest amount of gained bonus points against all the four basic types. We also ran the experiments for the case where more than 1 "row user" was present and the rational strategy always had superior performance compared to the other basic strategies.

In general, we conclude that of the 4 main strategies rational is the best, followed by greedy, random, and altruistic. The pure greedy and pure altruistic strategies usually exhibit very poor performance.

The exceptions to the dominance of the rational strategy are against a population of pure greedy or pure altruistic. In a population of pure greedy users, all chains are of length 2, since the first user in the chain will take all but 1 point. Therefore, the first user should take as much as she can, while guaranteeing the chain length of 2, i.e., take all but 2 points. The greedy strategy which takes more than the rational strategy is therefore better.

In a population of pure altruistic users, all chains have their maximum length, since all users take only 1 point. For example, with 20 points remaining, a rational user will take 14 and the chain has length 6, which gives the expected bonus points of $6 \cdot 14p = 84p$. If the user takes, say 10 points, then the chain has length 10. In this case, the expected amount of gained of bonus points is $10 \cdot 10p = 100p$.

5. RELATED WORK

Golle *et al.* [4] propose an analyzed several micro-payment mechanism to encourage file sharing and reduce the prevalent free-rider problem. Crowcroft *et al.* [2] propose a pricing mechanism for mobile ad hoc network nodes as an incentive to forward network packages. Mannak *et al.* [5] have conducted a small user study on user's motivation to share resources in peer-to-peer networks. They found out that 50% of the questioned users would share more, if incentives, e.g., money, would be provided by the application.

Ratsimor et al. [6] propose a scheme similar to ours, but in their model, a user cannot affect his chance of being rewarded, for example, by choosing a different strategy, since the strategies are fixed for all users.

Garyfalos and Almeroth describe *Coupons* [1,3], an incentive scheme that gives users credit for forwarding information to other users in an ad hoc network. Again, users are forced to use the equivalent of our pure altruistic strategy, which we found to exhibit very poor performance.

6. CONCLUSION

In this paper, we have defined a general model for electronic coupon systems based on rewarding user participation with bonus points. We have derived an optimal strategy which indicates how many points a user should take for herself when passing the coupon onwards. We have also evaluated our optimal strategy against a number of other possible strategies and have shown that our optimal strategy is extremely robust and provides a strong incentive for users to follow our optimal strategy.

In our future work, we plan on investigating the sensitivity of the rational strategy against a more complex setting, i.e., larger and non-complete passing trees with passing of coupons determined by user mobility and interests.

7. REFERENCES

[1] K. C. Almeroth and A. Garyfalos. Coupons: Wide Scale Information Distribution for Wireless Ad Hoc Networks. In *Globecom*, pages 1655–1659, 2004.

[2] J. Crowcroft, R. J. Gibbens, F. P. Kelly, and S. Östring. Modeling incentives for collaboration in mobile ad hoc networks. *Performance Evaluation*, 57(4):427–439, 2004.

[3] A. Garyfalos and K. C. Almeroth. Coupon Based Incentive Systems and the Implications of Equilibrium Theory. In *Conference on E-Commerce Technology. Proceedings*, pages 213–220. 2004.

[4] P. Golle, K. Leyton-Brown, I. Mironov, and M. Lillibridge. Incentives for Sharing in Peer-to-Peer Networks. In *Workshop on Electronic Commerce*, pages 75–87, 2001.

[5] R. Mannak, H. de Ridder, and D. V. Keyson. The Human Side of Sharing in Peer-to-Peer Networks. In *European Union symposium on Ambient intelligence*, pages 59–64, 2004.

[6] O. Ratsimor, T. Finin, A. Joshi, and Y. Yesha. eNcentive: A Framework for Intelligent Marketing in Mobile Peer-To-Peer Environments. In *International conference on Electronic commerce*, pages 87–94, 2003.

[7] T. Straub and A. Heinemann. An Anonymous Bonus Point System For Mobile Commerce Based On Word-Of-Mouth Recommendation. In *ACM Symposium on Applied Computing*, pages 766–773, New York, NY, USA, 2004.